

## Homework #1, PHY 674, 25 August 1995

- (X1). Let  $M$  and  $N$  be sets (not empty) and  $f : M \rightarrow N$  be a map. Show that  $f$  is bijective if and only if there is a map  $g : N \rightarrow M$  such that the compositions  $f \circ g$  and  $g \circ f$  are the identities  $\text{id}_N$  and  $\text{id}_M$  of  $N$  and  $M$ . (4 points).
- (X2). Show that there is only one neutral (or unit) element in each group. (4 points).
- (X3). Show that for each element  $x \in G$ , there is only one inverse  $x^{-1}$ . (4 points).
- (X4). Is there a group with zero elements ? (1 point)
- (X5). Find all groups with one, two, three, four, and five elements and write down their multiplication tables. Which of these groups are Abelian ? Hint: Use Lagrange's theorem. (4 points).

**Due Date:**

**Friday, September 1st, 1995, 2 pm**

in class or in the green homework box inside the south entrance to Room 12.